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azimuth and collimation errors, and if your instrument be faultless, and the Greenwich catalogue perfect, have no error left except the mere error of observation, which, when fourteen wires are noted, is very trifling. There is only one reduction to the mean place for two observations. Some observations of my own, and some made by Mr. Hartnup at my request, lead me to think that in two fine nights, a catalogue of quick moving stars may be formed, in which an error of $0^s{\cdot}05$ would be somewhat unusual.

On the Weights to be given to the separate Results, and to the final Result, for Terrestrial Longitudes determined by the Observations of Transits of the Moon and Fixed Stars. By the Astronomer Royal.

This paper contains the investigations for the principal cases which occur in practice. The *weight* of any one observation is taken as the reciprocal of the square of the probable error in seconds of time. The combination-weight of an observation is the number by which it is to be multiplied, in combining it with the results of other observations.

Case 1. The transits of the moon and one star, on a single evening, are compared with the places in the *Nautical Almanac*, considered as perfectly accurate: to find the weight and probable error of the result.

To find the weight of the result;—Form the fraction whose numerator is the product, and whose denominator is the sum, of the weights of the moon-observation and the star-observation, multiply this fraction by $\left(\frac{1}{3600}\right)^2$: the product is the weight of the result.

Case 2. Transits of the moon and several stars, observed in one evening, are compared with the places in the *Nautical Almanac*, supposed accurate.

The combination-weights for the several results will be the weights of the star-observations, without respect to the observation of the moon.

To obtain the weight of the final result;—Form the fraction whose numerator is the product of the weight of the moon-observation by the sum of the weights of the star-observations, and whose denominator is the sum of the weights of the moon-observation and the star-observation; multiply this fraction by $\left(\frac{1}{3600}\right)^2$: the product is the weight of the final result.

It will easily be seen, from this rule, that very little is gained by increasing the number of *stars* observed on the same night.

Case 3. Transits of the moon and several stars are observed at the primary station, and transits of the moon and several stars (it matters not whether the number of stars is the same, or whether

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the stars are identical) are observed at other stations on the same night; the tabular places of the stars are supposed perfectly accurate, but those of the moon are liable to error: to find the combination-weights and the weight of the result.

For the determined right ascension at each station: the combination-weight for the result as to moon's right ascension determined from each star will be the weight for that star, without respect to the observation of the moon. But the weight of the result for moon's right ascension determined at that station will be the fraction whose numerator is the product of the moon's weight by the sum of the weights of the stars, and whose denominator is the sum of the weights of the moon and the stars.

When the weight for each station is thus found, then the combination-weight to be used for the result for terrestrial longitude, inferred from comparison of the moon's right ascension determined at the primary station with the moon's right ascension determined at a comparison-station, is to be the weight for that comparison-station (as found in the preceding paragraph), without respect to the primary station. But the weight of the final result for terrestrial longitude will be found by forming the fraction whose numerator is the product of the weight for the primary station by the sum of the weights for all the comparison-stations, and whose denominator is the sum of the weights for the primary station and all the comparison-stations, and multiplying this fraction by $\left(\frac{1}{3600}\right)^2$, the product is the weight of the final result.

The following cases suppose no reliance on tables, either of the moon or stars:—

Case 4. Transits of the moon and one star at the primary station are compared with transits of the same objects at one comparison-station.

The weight of the result is the reciprocal of the sum of the reciprocals of the weights of the four observations, multiplied by $\left(\frac{1}{3600}\right)^2$.

Case 5. Transits of the moon and several stars observed at the primary station are compared with transits of the same objects at one comparison-station.

The combination-weights, for combining the results of the comparisons with the different stars, will depend only on the weights of observations of the stars, without respect to the observations of the moon.

The combination-weight for the result derived from comparisons with one star, will be the fraction whose numerator is the product of the two weights for that star, and whose denominator is the sum of those weights.

Suppose a similar weight to be formed in the same way from the moon-observations.

Then for the weight of the final result, form the fraction whose numerator is the product of the moon-weight, by the

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sum of the star-weights, and whose denominator is the sum of the moon-weight and the star-weights, multiply this fraction by $\left(\frac{1}{3600}\right)^2$, the product is the weight of the final result.

Case 6. Transits of the moon and a single star are observed at the primary station, and at several comparison-stations.

The combination-weights for the results derived from comparison with the different stations will depend only on the weights of the observations of the moon and star at the comparison-stations, without respect to those at the primary station.

The combination-weight for each comparison-station will be the fraction whose numerator is the product of the weights of the moon and the star, and whose denominator is their sum.

Suppose a similar weight to be formed for the primary station.

Then the weight of the final result is formed by forming the fraction, whose numerator is the product of the primary-station weight by the sum of the comparison-station-weights, and whose denominator is the sum of the primary-station weights and comparison-station weights: and multiplying this fraction by $\left(\frac{1}{3600}\right)^2$, the product is the weight of the final result.

Case 7. Transits of the moon and several (e) stars are observed at several (f) stations, besides the primary station on the same evening, the same stars being observed at all.

The general case having been treated, the particular case is fixed on in which all the moon-observations have equal probable errors (each $= \mu$), and also all the star-observations (each $= \pi$). The weight of the result is, then,

$$\left(\frac{1}{3600}\right)^2 \mu \times \text{reciprocal of } \left(1 + \frac{1}{f} + \frac{1}{e}\left(1 + \frac{1}{f}\right)\frac{\mu}{\pi}\right).$$

Case 8. As an instance of the case in which all the stars are not observed at all the stations, the following is taken.

At the primary station five stars are observed with the moon. Of these, the 1st, 2d, and 3d, are also observed with the moon at the first comparison-station; the 1st and 4th at the second comparison-station; and the 1st, 4th, and 5th, at the third comparison-station. All the observations of the moon are supposed equally good, and all the observations of the stars are supposed equally good.

The general formulæ having been considered, it is supposed that the moon and star-observations are equally good; and then, V, W, X, Y, Z, referring to the five stars, and the subscript numbers to the stations, the combination-weights V_1, W_1, X_1, V_2, Y_2, V_3, Y_3, Z_3, are found to be in the proportions of the numbers 33, 100, 100, 85, 96, 49, 60, 108. The weight of the final result is,—

$$\cdot 5930 \times \left(\frac{1}{3600}\right)^2 \times \mu.$$

The individual results receive smaller combination-weights

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when derived from those stations with which there is the greater number of comparisons. Again, the individual results receive smaller combination-weights when derived from comparisons made by means of stars observed at more stations. And, to illustrate the smallness of the advantage (compared with what would probably be expected) gained by increasing the number of stations and of comparisons, the probable error of the result in the above case is less than that of one comparison with one station, only in the proportion of 20 to 13.

In practice, approximate expressions for the weights, sufficiently near the truth, may be found by the following process. Set down in the form of a table of double entry, whose arguments are "stars observed" and "comparison-stations," points or marks indicating the observation. For every observation at a comparison-station where 1, or 2, or 3, or 4, &c. stars have been observed, set down the fraction $\frac{1}{2}$, or $\frac{1}{3}$, or $\frac{1}{4}$, or $\frac{1}{5}$, &c. For every observation of a star which has been observed at 1, or 2, or 3, &c. comparison-stations, set down the fraction $\frac{1}{2}$, or $\frac{1}{3}$, or $\frac{1}{4}$, &c. The products of these fractions will give nearly enough the weight for each comparison, and the sum of all the products multiplied by $\left(\frac{1}{3600}\right)^2 \times \mu$, will give nearly enough the weight of the whole. Thus, in the instance above:—

		Stars Observed.																			
		1st				2d				3d				4th				5th			
		Observed.	Station Number.	Star Number.	Product, or Weight.	Observed.	Station Number.	Star Number.	Product, or Weight.	Observed.	Station Number.	Star Number.	Product, or Weight.	Observed.	Station Number.	Star Number.	Product, or Weight.	Observed.	Station Number.	Star Number.	Product, or Weight.
Comparison-Stations.	1	*	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{16}$	*	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$	*	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$								
	2	*	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{12}$									*	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{9}$				
	3	*	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{16}$									*	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{12}$	*	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$

The numbers thus obtained are rather less unequal than those found by the accurate investigation; and the weight of the final result, $\mu \times \left(\frac{1}{3600}\right)^2 \times \frac{7}{9}$, is rather greater than that found by the accurate investigation. But as the rule answers exactly in extreme cases (whether the number of stations and the number of stars, or either of them, be very great or very small), it cannot err much in intermediate cases. It would be sensibly erroneous if a single different star were observed at every different station; but in this case the independent investigation would be very easy.

Case 9. Observations of any or several of the preceding classes are made *on different evenings.*

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The combination-weights for the results of different evenings are to be the weights found by the preceding investigations. And the weight of the final result will be the sum of these weights.

The attention of observers is particularly called to the circumstance that this is the *only* combination by which the weight of results is greatly increased. In all the preceding cases, however much the number of stars or number of comparison-stations was increased, the weight of the result could not be increased beyond a value very little greater than that for a single star at a single comparison-station. But by increasing the number of nights of observation, the weight of the final result may be increased without limit.

Measures of ε Böotis, by Mr. Fletcher.

"By a mean of four nights' measures of this star, taken under very favourable circumstances with the 6-foot equatoreal, I find,

Position 321° 40′. Distance 2″·93. Epoch 1850·41.

Note from Professor Piazzi Smyth.

"A careful examination of the heavens in a clear night has satisfied me that the stars in Lalande, 33239 and 33244, are the same. I find the following mean places with the transit and mural circle, Epoch 1850·48 :—

	Mag.	R.A. h m s	Obs.	N.P.D. ° ′ ″	Obs.	Epoch.
Lalande {33239 33244}	7	17 55 50·68	(3)	15 24 29·1	(3)	1850·48

On the Comet seen by Mr. Jenkins in November last.

In the *Monthly Notice* of March last, some account is given of a comet seen by Mr. Jenkins on his voyage to Rio Janeiro. It was supposed at the time that the comet thus seen might be the comet of long period which was expected in 1848, and further inquiry was made to ascertain the particulars more accurately.

Mr. Jenkins unfortunately died at Rio, but Captain Horner, the master of the Maryland (the vessel in which Mr. Jenkins sailed), states that the comet was seen on Nov. 15th, sea account: that the true time on board was $7^{h}\ 30^{m}$ P.M., latitude 13° 32′ South, long. 34° 50′ West.

The bearing of the comet from the ship was W.N.W., altitude of the nucleus 48°, and its course S.E. and N.W.; it was in sight about an hour.

Mr. Hind remarks that these data differ considerably from those previously received. The place of the comet, according to Captain Horner's particulars, would be,

R.A. $20^{h}\ 36^{m}\cdot 6$. N.P.D. 85° 42′ Nov. 15, at $9^{h}\ 49^{m}$ G.M.T.